N15/5/MATHL/HP3/ENG/TZ0/DM

Mathematics
Higher level
Paper 3 – discrete mathematics

Wednesday 18 November 2015 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.

5 pages

8815 – 7207

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 11]

(a) The distances by road, in kilometres, between towns in Switzerland are shown in the following table.

	Basel	**Berne**	**Geneva**	**Lugano**	**Sion**	**Zurich**
Basel		100	260	260	250	85
Berne	100		170	275	155	125
Geneva	260	170		440	160	290
Lugano	260	275	440		255	210
Sion	250	155	160	255		275
Zurich	85	125	290	210	275	

A cable television company wishes to connect the six towns placing cables along the road system.

Use Kruskal's algorithm to find the minimum length of cable needed to connect the six towns. [5]

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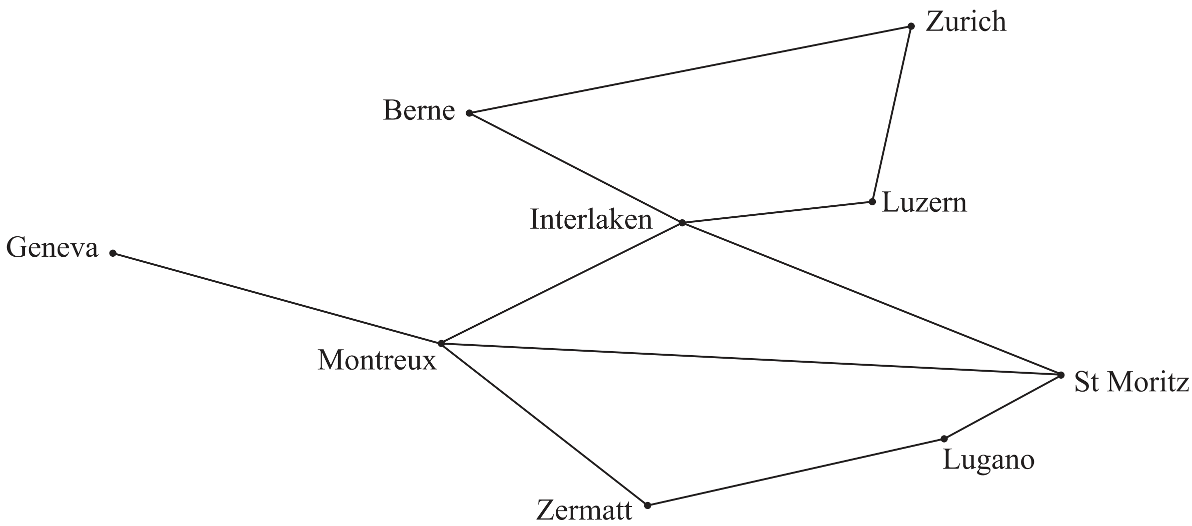
(Question 1 continued)

(b) Visitors to Switzerland can visit some principal locations for tourism by using a network of scenic railways as represented by the following graph:



(i) State whether the graph has any Hamiltonian paths or Hamiltonian cycles, justifying your answers.

(ii) State whether the graph has any Eulerian trails or Eulerian circuits, justifying your answers.

(iii) The tourist board would like to make it possible to arrive in Geneva, travel all the available scenic railways, exactly once, and depart from Zurich. Find which locations would need to be connected by a further scenic railway in order to make this possible. [6]

2. [Maximum mark: 13]

A recurrence relation is given by $u_{n+1} + 2u_n + 1 = 0$, $u_1 = 4$.

(a) Use the recurrence relation to find u_2. [1]

(b) Find an expression for u_n in terms of n. [6]

A second recurrence relation, where $v_1 = u_1$ and $v_2 = u_2$, is given by $v_{n+1} + 2v_n + v_{n-1} = 0$, $n \geq 2$.

(c) Find an expression for v_n in terms of n. [6]

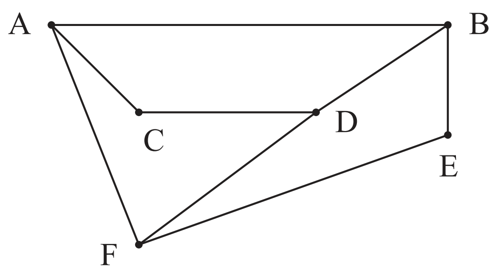
3. [Maximum mark: 13]

(a) Show that there are exactly two solutions to the equation $1982 = 36a + 74b$, with $a, b \in \mathbb{N}$. [8]

(b) Hence, or otherwise, find the remainder when 1982^{1982} is divided by 37. [5]

4. [Maximum mark: 13]

The following diagram shows the graph G.



(a) Show that G is bipartite. [2]

(b) State which two vertices should be joined to make G equivalent to $K_{3,3}$. [1]

In a planar graph the degree of a face is defined as the number of edges adjacent to that face.

(c) (i) Write down the degree of each of the four faces of G.

(ii) Explain why the sum of the degrees of all the faces is twice the number of edges. [2]

H is a simple connected planar bipartite graph with e edges, f faces, v vertices and $v \geq 3$.

(d) Explain why there can be no face in H of degree

(i) one;

(ii) two;

(iii) three. [3]

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(Question 4 continued)

(e) Hence prove that for H

(i) $e \geq 2f$;

(ii) $e \leq 2v - 4$. [3]

(f) Hence prove that $K_{3,3}$ is not planar. [2]

5. [Maximum mark: 10]

(a) Given a sequence of non negative integers $\{a_r\}$ show that

(i) $\sum_{r=0}^{n} a_r (x+1)^r (\text{mod}\, x) = \sum_{r=0}^{n} a_r (\text{mod}\, x)$ where $x \in \{2, 3, 4...\}$.

(ii) $\sum_{r=0}^{n} \left(3a_{2r+1} + a_{2r}\right) 9^r = \sum_{r=0}^{2n+1} a_r 3^r$. [5]

(b) Hence determine whether the base 3 number 22010112200201 is divisible by 8. [5]